André Weil (1906-98)

One of the Century's most influential pure mathematicians

André Weil, number-theorist and geometer, died at his home in Princeton, New Jersey, on the 6 August. Born in Paris in 1906, he quickly manifested precocious mathematical and philological interests. His gifts and those of his younger sister, the philosopher Simone Weil, were fostered by a watchful, vigorous mother, so that, after several years of the best Parisian *lycées* and excellent tutors, he entered the *École normale supérieure* at the early age of sixteen. Small of stature, light of build, and myopic, he remained until very late in life somewhat of an *enfant terrible*, competitive, a little vain, overbearing with colleagues, but with great resources of charm and culture and an intellectual eagerness that never flagged.

While at the ENS he was admitted to the seminar of Jacques Hadamard, whose style – a wide-ranging, penetrating curiosity about all branches of mathematics – Weil attempted to make his own, very largely succeeding. But his thesis, published when he was twenty-two, was suggested, by his own account, not by the seminar but by his study of the masters of the past, especially of Fermat and Riemann. On number theory, the study of solutions of algebraic equations either in whole numbers or in fractions, the thesis contained among other things a theorem, the Mordell-Weil theorem, that remains to this day of central importance.

Meanwhile, convinced that with the exception of Hadamard and Élie Cartan his professors in Paris were out of touch with recent mathematics, he travelled, first to Italy and then to Germany. There, at a time when the mathematical world was much more diverse, both scientifically and linguistically, than today, he could become familiar with the algebraic geometry of the Italians and the number theory of the Germans. His greatest achievement was to be the infusion of number theory with modern geometric notions and the concomitant reworking, by him and by those he influenced, of the foundations of algebraic geometry.

When investigating solutions in whole numbers of an algebraic equation, or of several simultaneous equations, the mathematician's first step is often to replace the equations by congruences. This

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means to search for whole numbers that render the relevant expressions divisible by some given number, usually prime. The number of solutions of any given collection of congruences can be counted. It had been suggested, and proved in some cases, that, for congruences defined by one equation in two unknowns and a given prime, this number (together with the number of solutions of related congruences) could be used to form a function with properties similar to those believed to be possessed by the zeta-function of Riemann, a function of a complex variable. The Riemann hypothesis, which concerns the points at which this function is zero, has been for more than a century the most important unsolved problem of mathematics. But there is an easier Riemann hypothesis for congruence zeta-functions, which yields universal estimates of the number of solutions to the congruences.

Weil succeeded in establishing that hypothesis in unusual personal circumstances. A tourist in Finland at the outbreak of the Second World War, Weil chose to remain there and not to return home to fulfil his military obligations. After the Russian invasion of Finland, he was arrested as a suspected spy. Thanks to a prominent Finnish mathematician, he was not summarily executed but deported to Sweden, where he was taken into custody and returned to France. There, he spent several months in a Rouen prison before being convicted of failure to report for duty. He agreed to serve in a combat unit and the sentence was suspended.

In prison he drafted the main lines of his proof and, circumstances being what they were, quickly published them, but several years elapsed before he completed the prerequisite treatise on the foundations of algebraic geometry, and the reworking, in the context of congruences, of the classical theory of Abelian integrals and the attendant topological notions on which the proof was based.

These investigations were completed in the United States and in Brazil, where, his combat unit having earlier been evacuated to England after the Franco-German armistice, he spent the war years with his wife and family. Not long after the war, in Chicago, idly reading the papers of Gauss, Weil was led to the general conjectures on congruences and zeta-functions that bear his name. These conjectures have since been demonstrated by Alexandre Grothendieck and his school, and their influence, on Weil himself, on Grothendieck, and on others, has permeated number theory and algebraic geometry.

The initial impulse to the recent proof of Fermat's theorem was the insight that it was a consequence of a conjecture of Taniyama, itself prompted by a conjecture of Hasse and Weil. The ideas and techniques that were then used in the proof of Fermat's theorem were many and varied and arose from a multitude of often distant sources, but Andrew Wiles's achievement is unimaginable without notions central to the proof of the Weil conjectures.

There is no question of doing justice here to Weil's manifold contributions to mathematics, but his influence was by no means a result of his theorems alone. A rich and solid, although occasionally strained, prose style in both French and English won a wide audience for his strong views about the nature and the teaching of mathematics.

After more than two years in India, to which an early infatuation with Sanskrit had perhaps drawn him, Weil had returned in 1932 to France, where he and several friends, dissatisfied with the calculus texts available, undertook a collective project that gave birth to an unusual mathematical personality – Nicolas Bourbaki, articulate advocate of a specifically French view of mathematics, author of a widely read treatise in many volumes on its elements, and founder of a seminar that continues to this day.

In 1958 Weil joined the Faculty of the Institute for Advanced Study, where he was active until very few years before his death, as a mathematician and later as an historian of mathematics. His memoirs, written after the death of his wife, to whom he had been deeply attached, throw considerable light on his character.